# AMRITA VIDYALAYAM

## AMRITA PRE BOARD EXAMINATION 2017 -'18

**Class : XII** **Marks : 100**

**Time : 3 hrs**

## MATHEMATICS

***GENERAL INSTRUCTIONS:***

***1. All questions are compulsory.***

***2. This question paper consists of 29 questions divided into four sections A, B, C and D.***
***Section A comprises of 4 questions of 1 mark each.***
***Section B comprises of 8 questions of 2 marks each.***
***Section C comprises of 11 questions of 4 marks each.***
***Section D comprises of 6 questions of 6 marks each.***

***3. All questions in section A are to be answered in one word, one sentence or as per exact requirement of the question.***

***4. There is no overall choice. However, an internal choice has been provided in 3 questions of 4 marks each and 3 questions of 6 marks each.***

***5. Use of calculator is not permitted.***

## SECTION - A

1. Find the value of x, y, z if

$$\begin{bmatrix} 2x+3 & y-2 \\ 3z-2 & 3 \end{bmatrix} = \begin{bmatrix} 7 & 4 \\ 10 & 3 \end{bmatrix}$$

2. Find the derivative of $\sin(x^2+1)$.
3. Evaluate $\int (\sqrt{x} - 1/\sqrt{x})^2 dx$.
4. Write the direction cosines of the normal to the plane $3x + 4y + 12z = 0$.

## SECTION - B

5. If $\begin{bmatrix} 2x & 5 \\ 8 & x \end{bmatrix} = \begin{bmatrix} 6 & -2 \\ 7 & 3 \end{bmatrix}$

   Find the value of x.
6. Examine the continuity of the function.

$$F(x) = \left\{ \begin{array}{ll} x^2 - 1 / x - 1 & \text{for } x \neq 1 \\ 2 & \text{for } x = 1 \end{array} \right\}$$

   Show that f(x) is continuous at x = 1.
7. The volume of a cube is increasing at the rate of $9 \text{ cm}^3/\text{s}$. How fast is its surface area increasing when the length of an edge is 10 cm?
8. Show that the function $f(x) = x - (1/x)$ is a strictly increasing function for all $x > 0$.
9. Find the vector equation for the line passing through the points (-1, 0, 2) and (3, 4, 6).
10. Two dice are thrown. Find the probability of getting an odd number on the first die and multiple of 3 on the other.
11. Solve the following LPP graphically.

    Maximize $z = 3x + 4y$ subject to the constraints $x + y \leq 4$, $x \geq 0$, $y \geq 0$.
12. Evaluate $\int 1 - \sin x / \cos^2 x \, dx$.

## SECTION - C

13. Prove that

    $\cot^{-1} 7 + \cot^{-1} 8 + \cot^{-1} 18 = \cot^{-1} 3$.

OR

If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi/2$ then show that $xy + yz + zx = 1$.

14. Solve for x $\begin{vmatrix} x+2 & x+6 & x-1 \\ x+6 & x-1 & x+2 \\ x-1 & x+2 & x+6 \end{vmatrix} = 0$

OR

If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 2 & -3 \end{bmatrix}$

Verify that $(AB)^I = B^I A^I$

15. If $y = e^{a \cos^{-1} x}$ show that $(1 - x^2)\, d^2y/dx^2 - x\, dy/dx - a^2 y = 0$.

16. Evaluate $\int \dfrac{3x - 2}{(x + 3)(x + 1)^2}\, dx$.

17. Prove that $\int_0^{\pi/2} \dfrac{\sin x}{\sin x + \cos x}\, dx$.

OR

Evaluate the integral $\int_0^2 (x + 4)\, dx$ as a limit of a sum.

18. Form the differential equation of the family of circles having radii 3.

19. Find the area of a triangle whose vertices are A (-1, -2, -3) B (2, 0, 3) and C (4, 6, 0).

20. Show that the four points (0, -1, -1), (4, 5, 1), (3, 9, 4) and (-4, 4, 4) are coplanar. Also find the equation of plane containing them.

21. A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

22. A retired person wishes to invest a sum of 12,000 in two types of investments in a mutual fund which provides 10% per annum and 8% in national saving bonds. According to the rule he has to invest minimum of 2,000 in mutual funds and 4,000 in national savings. How much should he invest to earn a minimum yearly income? What is the advantage of investing in national saving bonds?

23. An insurance company insured 2,000 scooter drivers 4,000 car drivers and 6,000 truck drivers. The probability of an accident are 0.01, 0.03, and 0.15 respectively. One of the insured persons meet with an accident. What is the probability that he is a scooter driver?

## SECTION - D

24. If $A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{bmatrix}$

Find $A^{-1}$ and hence solve the following system of equations.

$2x + y + 3z = 3$

$4x - y = 3$

$-7x + 2y + z = 2$

25. Let A = {1, 2, 3, …..9} and R be the relation in $A \times A$ defined by (a, b) R (c, d), if a + d = b + c. Show that R is an equivalence relation. Also obtain the equivalence class [(2, 5)].

OR

Let f: $N \rightarrow R$ be a function defined as $f(x) = 4x^2 + 12x + 15$. Show that $f: N \rightarrow S$ is invertible where S is the range of f. Hence find inverse of f.

26. A square piece of tin of side 18cm is to be made into a box without corner and folding up the flaps to form the box. What should be the side of square to be cut off so that the volume of the box is the maximum?

27. Prove that the curves $y^2 = 4x$ and $x^2 = 4y$ divide the area of the square bounded by x = 0, x = 4, y = 4 and z = 0 into three equal parts.

OR

If the area bounded by the parabola $y^2 = 16ax$ and the line $y = 4mx$ is $a^2/12$ sq units, find the value of m.

28. Find a particular solution of the differential equation $2y\, e^{x/y}\, dx + (y - 2xe^{x/y})dy = 0$ given that $x = 0$ when $y = 1$.

29. Find the equation of the plane passing through the point (6, 5, 9) and parallel to the plane determined by the points A(3, -1, 2) B(5, 2, 4) and C (-1, -1, 6). Also find the distance of the plane from the point A.

OR

Find the image of the point having the position vector $\hat{i} + 3\hat{j} + 4\hat{k}$ in the plane $\vec{r}.(2\hat{i} - \hat{j} + \hat{k}) + 3 = 0$.