AMRITA VIDYALAYAM

AMRITA PRE BOARD EXAMINATION 1 - 2018 - '19

Class : XII **Marks : 100**

Time : 3 hrs

MATHEMATICS

General instructions

1. ***All questions are compulsory.***
2. ***This question paper consists of 29 questions divided into four section A, B, C and D.***
   Section A comprises of 4 questions of 1 mark each.
   Section B comprises of 8 questions of 2 marks each.
   Section C comprises of 11 questions of 4 marks each.
   Section D comprises of 6 questions of 6 marks each.
   All questions in section A are to be answered in one word, one sentence or as per the exact requirement of the question.
3. ***There is no overall choice. However, an internal choice has been provided in all sections***
4. ***Use of calculator is not permitted.***

SECTION - A

1. If $A^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$ then find $A^T - B^T$.
2. Evaluate $\int \frac{\log x}{x}\, dx$ or $\int \tan^2 x\, dx$.
3. If the cartesian equation of a line is $\frac{3 - x}{5} = \frac{y + 4}{7} = \frac{2z - 6}{4}$ write the vector equation for the line.
4. Write the sum of the order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 = \left(\frac{dy}{dx}\right)^3 + x^4 = 0$.

SECTION - B

5. Prove that $\tan\left(\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{a}{b}\right) + \tan\left(\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\frac{a}{b}\right) = \left(\frac{2b}{a}\right)$
6. Find the unit vector in the direction of the sum of the vectors $2\hat{i} + 3\hat{j} - \hat{k}$ and $4\hat{i} - 3\hat{j} + 2\hat{k}$.
7. Let A = $z \times z$ and * be binary operation on A defined by $(a, b) * (c, d) = (ad + bc, bd)$. Find the identify element for * in the set A.

   OR

   Prove that the function f: R $\rightarrow$ R defined as f(x) = $2x - 3$ is invertable. Also find f^{-1}.
8. Find the equation of the tangent to the curve $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point $(\sqrt{2}\,a, b)$.
9. Solve. $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$
10. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ find k so that $A^2 = 5A + KI$.
11. If $\tan^{-1}x + \tan^{-1}y = \pi/4$, $xy < 1$ then write the value of $x + y + xy$.
12. Express $\begin{bmatrix} 5 & 1 \\ 2 & -3 \end{bmatrix}$ as the sum of symmetric and skew symmetric matrices.

    OR

    Using elementary row operations, find the inverse of the following matrix $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$.

SECTION - C

13. Show that the relation S in the set A = $\{x \in z : 0 \le x \le 12)$ given by S = $\{(a,b) : a, b \in z\ |a - b|$ is divisible by 4} is an equivalance relation.
14. Prove that following using properties of determinants.

$$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$$

15. Using matrices solve the following system of equations.

x + 2y - 3z = -4
2x + 3y + 2z = 2
3x - 3y - 4z = 11

16. If $y = (\tan^{-1}x)^2$

Show that $(x^2 + 1)^2 \frac{d^2y}{dx^2} + 2x\ (x^2+1)\frac{dy}{dx} = 2$

OR

If $x^y + y^x = a^b$ then find $\frac{dy}{dx}$.

17. Find the value of 'a' for which the function defined by

$f(x) = a \sin\frac{\pi}{2}(x+1)$ if $x \le 0$

$= \frac{\tan x - \sin x}{x^3}$, if $x > 0$

is continuous at $x = 0$

18. Find the intervals in which the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ is

a) strictly increasing. b) strictly decreasing.

OR

Find the appropriate value of $f(3.02)$ upto 2 places of decimal, where $f(x) = 3x^2 + 5x + 3$.

19. $\int \frac{2x}{(x^2+1)\ (x^4+4)}\,dx$

OR

$\int \frac{6x+7}{\sqrt{(x-5)\ (x-4)}}\,dx$

20. Using vectors, find the area of the triangle with vertices A (1, 1, 2) and B (2, 3, 5) and C (1, 5, 5).

OR

If vectors $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$ are such that $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$, then find the value of λ.

21. Solve the differential equation.

$y + x\frac{dy}{dx} = x - y\frac{dy}{dx}$

OR

Solve $\frac{dy}{dx} + 2y \tan x = \sin x$ given that $y = 0$ when $x = \frac{\pi}{3}$.

22. Four coins are tossed simultaneously. What is the probability of getting

a) 2 heads? b) at least 2 heads?

23. Find the co-ordinates of the point when the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{4}$ intersects the plane $x - y + z - 5 = 0$. Also find the angle between the line and plane.

SECTION - D

24. Find the equation of the plane passing through the points (-1, 2, 0), (2, 2, -1) and parallel to the line $\frac{x-1}{1} = \frac{2y+1}{2} = \frac{z+1}{-1}$

OR

Find the equation of a line passing through the point (1, 2, -4) and perpendicular to two lines

$\vec{r} = 8\vec{i} - 19\vec{j} + 10\vec{k} + \lambda(3\hat{i} - 16\hat{j} + 7\hat{k})$

and $\vec{r} = 15\vec{i} + 29\vec{j} + 5\vec{k} + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$.

25. Evaluate $\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9 + 16\sin 2x}\, dx$

OR

$\int_0^{\frac{\pi}{2}} \log \sin x \; dx$

26. A window is in the form of a rectangle surrounded by a semi-circular opening. The total perimeter of the window is 10*m*. Find the dimensions of the rectangle so as to admit maximum light through the whole opening.
27. Using the method of integration, find the area of the region bounded by the lines $3x - 2y + 1 = 0$, $2x + 3y - 21 = 0$ and $x - 5y + 9 = 0$.
28. A society of farmers has 50 hectares of land to grow 2 crops A and B. The profits from crops A and B per hectare are ` 10,500 and ` 9,000 respectively. To control weeds, a herbicide has to be used for A and B at the rate of 20 litres and 10 litres per hectare respectively. Further not more than 800 litres of herbicide should be used to protect the environment. How much land should be allocated to each crop so as to maximize the total profit? Formulate an LLP and solve it graphically. Do you agree that protection of wildlife is necessary to preserve the balance in environment? Justify.
29. A doctor is to visit a patient. From the past experience, it is known that the probabilities that he will come by train, bus, scooter or by other means of transport are respectively $\frac{3}{10}$, $\frac{1}{5}$, $\frac{1}{10}$ and $\frac{2}{5}$. The probabilities that he will be late are $\frac{1}{4}$, $\frac{1}{3}$ and $\frac{1}{12}$ if he comes by train, bus and scooter respectively, but if he comes by other means of transport then he will not be late. When he arrived, he is late. What is the probability that he came by train?