AMRITA VIDYALAYAM

FIRST TERMINAL EXAMINATION 2018 -'19

Class : XII **Marks : 100**

Time : 3 hrs

MATHEMATICS

GENERAL INSTRUCTIONS:

1. All question are compulsory
2. Question 1-4 in Section A are carrying 1 mark each.
3. Question 5-12 in section B are carrying 2 marks each
4. Question 13-23 in Section C are carrying 4 marks each.
5. Question 24-29 in Section D are carrying 6 marks each.

SECTION - A

1. Find the principal value of $\cos^{-1}(\cos 7\pi/6)$
2. If $f(x) = x + 7$ and $g(x) = x - 7$, find $f \circ g(7)$.
3. Find the value of x if $\begin{bmatrix} 5x + y & -y \\ 2y - x & 3 \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ -3 & 3 \end{bmatrix}$
4. A is a square matrix and $|A| = 2$ then write the value of $|AA^T|$.

SECTION - B

5. Find the vector and Cartesian equation for the line through the points (3, 4, -7) and (5, 1, 6).
6. Using differential find the approximate value of $\sqrt{49.5}$.

   OR

   Find the approximate change in the volume V of a cube of side x metres caused by increasing the side by 2%.
7. Using Row transformation find the inverse of matrix $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$
8. Solve $\sin \sin^{-1}(1 - x) - 2\sin^{-1} x = \frac{\pi}{2}$.
9. Prove that $\cot^{-1} 7 + \cot^{-1} 8 + \cot^{-1} 18 = \cot^{-1} 3$.
10. Find x and y if $x + y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ $x - y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$
11. Find the angle between the pair of lines.

    $\frac{-x + 2}{-2} = \frac{y - 1}{7} = \frac{z + 3}{-3}$ and $\frac{x + 2}{-1} = \frac{2y - 8}{4} = \frac{z - 5}{4}$
12. Verify Rolles theorem for the function.
    $f(x) = x^2 - 4x + 3$ on (1, 3).

SECTION - C

13. Consider $f : R + \rightarrow [-5, \alpha)$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible. Hence find $f^{-1}(10)$.
14. If $x = a\cos^3\theta$ $y = a\sin^3\theta$ then show that the value of $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{6}$ is $\frac{32}{27a}$.
15. Using properties of determinants prove the following.

$$\begin{vmatrix} x & x^2 & 1 + px^3 \\ y & y^2 & 1 + py^3 \\ z & z^2 & 1 + pz^3 \end{vmatrix} = (1 + pxyz)(x - y)(y - z)(z - x)$$

16. Differentiate the following function.
$(x)^{\cos x} + (\sin x)^{\tan x}$

OR

If $y = \cos^{-1} x$. Find $\frac{d^2y}{dx^2}$ in terms of y alone.

17. If $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$ find $A^2 - 5A + 4I$.

18. Find the intervals in which the function f give in by $f(x) = x^3 + \frac{1}{x^3}$, $x \neq 0$ is strictly

a) increasing. b) decreasing.

OR

Find the equations of the tangent and normal to the curve.
$x^{2/3} + y^{2/3} = 2$ at (1, 1)

19. Vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are such that $\vec{a} + \vec{b} + \vec{c} = 0$ and $|\vec{a}| = 3$ $|\vec{b}| = 5$ and $|\vec{c}| = 7$. Find the angle between $\vec{a}$ and $\vec{b}$.

20. Find the shortest between the lines whose vector equations are
$\vec{r} = \hat{i} + \hat{j} + \gamma(2\hat{i} - \hat{j} + \hat{k})$
$\vec{r} = 2\hat{i} + \hat{j} - \hat{k} + \mu(3\hat{i} - 5\hat{j} + 2\hat{k})$

21. Find the value of 'P' when the curves $x^2 = 9p(9 - y)$ and $= x^2 = p(y + 1)$. Cut each other at right angles.

22. If $f(x) = \frac{\sin(a+1)x + \sin x}{x}$, $x < 0$
$= C$, $x = 0$
$= \frac{\sqrt{x + bx^2} - \sqrt{x}}{bx\sqrt{x}}$, $x > 0$

23. Find the equation of the plane passing through the line of intersection of the planes $2x + y - z = 3$ and $5x - 3y + 4z = -9$ and parallel to the line.
$\frac{x-1}{2} - \frac{y-3}{4} = \frac{z-5}{5}$

SECTION - D

24. If $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$

Find AB and use if to solve the system of equations.
$x - y + z = 4$
$x - 2y - 2z = 9$
$2x + y + 3z = 1$

25. Find the image of the point (1, 3, 4) in the plane $2x - y + z + 3 = 0$.

26. Prove that the volume of the largest cone, that can be described in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

OR

S.T the semi vertical angle of a right circular cone of given surface area and maximum volume is $\sin^{-1}\left(\frac{1}{3}\right)$.

27. Let * be a binary operation defined on A = NxN such that
    (a, b) * (c, d) = (ad + bc, bd) $\forall$ (a, b), (c, d) ε A
    S.T * is commutative on A
    * Is Associative on A
    A has no identity element
28. a). If $\vec{a} = 3i - j$ and $\beta = 2i + j - 3k$ then express $\vec{\beta}$ in the form $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$
    where $\vec{\beta}_1$ parallel to α and $\vec{\beta}_2$ perpendicular to α.
    b) Find area of triangle having the points A (1, 1, 1) B (1, 2, 3) and C (2, 3, 1) as its vertices.
29. A manufacturing company makes two types of teaching aids A and B, of Maths for class XII. Each type of A requires 9 labour hours of fabricating and 1 labour hour for finishing. Each type of B requires 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing the maximum labour hours available per week are 180 and 30 respectively. The company makes a profit of ` 80 on each price of type A and ` 120 on each price of type B. How many pieces of type A and type B should be manufactured per week to get maximum profit? Formulate the problem as a LPP and solve it graphically. What is the maximum profit per week?